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Case Study: Core Network Design using Graph Theory Method

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Abstract— In recent years, due to the vast amount of information that is being exchanged between people and the diversity of the type of this information as well as the introduction of the Internet into the public domain in the beginning of the 1990s. The information revolution and the introduction of the Internet made many organizations think about utilizing these benefits for their own use, which ultimately lead to the introduction of the Intranet in the middle of the 1990s, which uses web technology within the boundaries of an organization,

This paper is concerned with finding the best method to connect the various faculties and other services in Tripoli University in order to implement an intranet using a unified switched Local Area network, based on the assumption that there exists an infrastructure of underground conduits suitable for laying fiber optical cables.

The paper assumed the use of single mode fiber cables to connect the various buildings in the core network.

In this paper the graph theory criteria, namely, the hop count, the average path length and the number of geodesics were used as bases to select the most suitable topology. But the result of the graph theory method was not decisive in selecting a topology from the three compared topologies (Star, Ring and Partial Mesh). Hence a management tool known as the Analytic Hierarchy Process (AHP) was employed to reach a decisive decision. The Partial Mesh topology was the selected topology according to the AHP result.

KEYWORDS: GRAPH THEORY, SWITCHED NETWORK, TOPOLOGY, PATH LENGTH, GEODESICS, HOP COUNT.

I. Introduction

In modern network design, the network infrastructure is usually divided to modules for ease of design and visualization. This modular approach known as the hierarchy model. The network is divided to three layers known as the core, distribution and access. These layers are not necessarily to be physically separate, but it is a separation for ease of visualization.

This paper is concerned with the core layer of Tripoli university section 'A' proposed network.

The work proceeded as follows:

- It is started by data collection through personal interviews with the head of the university engineering office and some of the staff to gather information on the existing facilities that can be utilized such as existing cable conduits running between the various faculties and the population for each faculty in the university's section 'A'.
- Based on the information collected, the nodes for the network were selected.
- A set of measures were applied to the most commonly used topologies namely Star, Ring and Partial Mesh based on graph theory to select the most appropriate alternative of connecting a number of nodes using the analytic hierarchy process method.

II. Selected Nodes

9 nodes were selected for the university section 'A' core network. Table (1) shows the list of the selected nodes and their abbreviations.

Table (1) Node Name abbreviation

Node name	Abbreviation
Veterinary	VET
Agriculture	AGR
Pharmacy & medical technology	PH&MT
Medicine	MED

Engineering	ENG
Nuclear & Electronics	N&E
Administration	ADM
Science	SCE
Law	LAW

III. Graph Theory Method

Computer networks are constructed to help humans exchange information and knowledge. Human's interaction is very complex system of interconnected system of nodes (people & groups) and ties (relationships and flows) and these map to computer networks.

Human networks are analyzed by a method known as social network analyses. This analysis is based on graph theory [1, 2].

In this theory a graph consists of a set of elements called nodes and a set of arcs called links. A graph can have two kinds of links directed and undirected. A directed link indicates that information can only flow in the direction indicated by the link. An undirected link indicates that information can flow both ways.

Undirected graph can be used to model full duplex channels, which can accept flow in either direction simultaneously or half duplex channels, which can accept flow in either direction but in only one direction at a given instant.

Definitions

- The degree of a node represents the number of links ending at that node. In a directed graph both incoming and outgoing links count. Due to Euler's theorem [2], the sum of the degrees of all nodes in a graph is equal to twice the number of links.
- A path is a sequence of adjacent nodes. The length of a path is the number of links it contains. The shortest path between a pair of nodes is called a geodesic.
- The diameter of a graph is the length of the longest geodesic.

Measuring the network location is finding the centrality of a node. The relationship between the centralities of all nodes in a network can reveal much about the overall network structure. One or few central nodes dominate a very centralized network. If these nodes fail, they will fragment the network to unconnected sub-networks. Also they can be a critical point of failure. [1]

There are three measures used to analyses networks:

- **Reducing hop count**: that is minimizing the average path length throughout the network, maximizes the closeness of all nodes to each other. In other words minimizes delay and maximizes inter node response time.
- **Reducing available paths**: leads to minimizing the number of geodesics throughout the network, which should result in an overall path distribution i.e. good connectivity without excessive routing tables.
- **Minimizing failures**: that is minimizing the centralization of the whole network, i.e. having enough redundancy to improve efficiency and reliability.

For Tripoli University core network, three of the most widely used network topologies have been considered and the above-mentioned measures have been applied to each topology.

IV. Selected Topologies

The selected topologies for this core network are: Star; Ring and Partial Mesh.

1. Star Topology

The Star topology, shown in figure (1), has many advantages. Among the many advantages of this topology, are ease of management and configuration for network administrators. But it has one glaring fault it is too central. Applying the previously outlined measures for this topology reveals the following:

• From tables (2 and 3), the average path length (1.78) throughout the network satisfies this goal well. Any node can reach any other node in 2 hops.

• From table (3) there are a total of 72 possible paths available, 16 paths of length 1 hop and 56 of length 2 hops, this will not overload the routing tables nor cause delays during routing table updates. It takes only 8 bi-directional links to create the available paths. [1]

• This network fails completely if node N&E fails, also any link fails isolates the attached node; there are no multiple paths to reach each node. Node N&E as well as being a single point of failure, it is also a potential bottle –neck.

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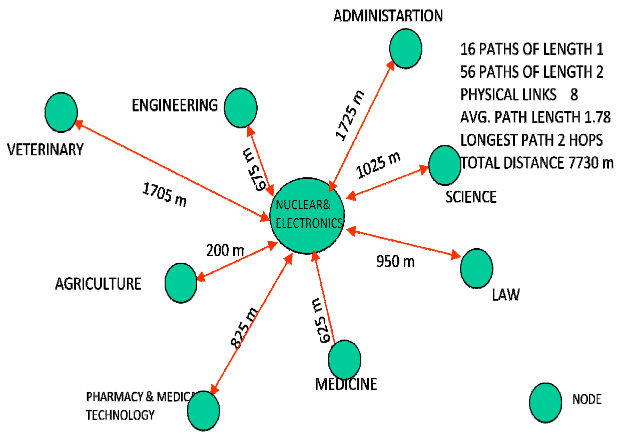

Figure (1) University Network Star Topology

It will likely become over burdened with packet flows and routing updates as more nodes are added.

Calculations of the average path length and the total number of geodesics, using the graph theory proceeds as follows:

1. Develop the path matrix between all the nodes, by counting the minimum number of hops between each node and all other nodes as shown in table (2).

2. Calculate the total number of geodesics: once the path matrix was developed, find the sum of each path length type for all nodes. In this example there are paths of length (1) and length (2), since the diameter of this network is 2. From table (3) above the paths of length (1) are 16 and paths of length (2) are 56 and hence the total number of geodesics is:

$$Paths\ of\ length\ (1) + paths\ of\ length\ (2) = 16 + 56 = 72 \qquad (1)$$

3. Calculate the average path length of the network:

$$Average\ path\ lengh = \frac{Paths\ lenght\ (1) \times 1 + Paths\ of\ length\ (2) \times 2}{Total\ number\ of\ geodesics}$$

$$= \frac{(16 \times 1 + 56 \times 2)}{72} = 1.78 \qquad (2)$$

Table (2) Path Matrix

Faculty	VET	AGR	PH & MT	MED	N&E	ENG	ADM	SCE	LAW
VET	0	2	2	2	1	2	2	2	2
AGR	2	0	2	2	1	2	2	2	2
PH&MT	2	2	0	2	1	2	2	2	2
MED	2	2	2	0	1	2	2	2	2
N&E	1	1	1	1	0	1	1	1	1
ENG	2	2	2	2	1	0	2	2	2
ADM	2	2	2	2	1	2	0	2	2
SCE	2	2	2	2	1	2	2	0	2
LAW	2	2	2	2	1	2	2	2	0

Table (3) Average path length and total number of geodesics

Faculty	L1	L2
VET	1	7
AGR	1	7
PH&MT	1	7
MED	1	7
N&E	1	0
ENG	1	7
ADM	1	7
SCE	1	7
LAW	1	7

L1 = Path of length 1

L2 = Path of length 2

Average path length = 1.78

If the number of links is doubled in this topology to increase throughput due to application demand, i.e. there will be 16 physical links, this increase will result in an increase in the total number of available paths to 256, which amounts to 255% increase in the available paths. This might overload the routing table and most certainly will cause some delay during routing table updates. Therefore due to the high centrality of this topology and taking into account that this core network will be used for an Intranet where 80% of traffic will propagate through the core, the network will be overloaded very quickly and it will be very hard to manage and reconfigure. [1, 4].

If dual Star topology is used to improve redundancy of the network then the measures for this modified Star topology will be:

Average path length = 1.8

Hop count (geodesics) = 160

Longest path = 2 hops

Total distance = 15460 m

From the above calculated measures, it can be seen that improving redundancy in the Star topology will increase average path by small amount, increase hop

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Table (4) Path Matrix

FACULTY	VET	AGR	PH&MT	MED	N&E	ENG	ADM	SCE	LAW
VET	0	1	2	3	4	1	3	2	4
AGR	1	0	1	2	3	2	4	3	4
PH&MT	2	1	0	1	2	3	4	4	3
MED	3	2	1	0	1	4	3	4	2
N&E	4	3	2	1	0	4	2	3	1
ENG	1	2	3	4	4	0	2	1	3
ADM	3	4	4	3	2	2	0	1	1
SCE	2	3	4	4	3	1	1	0	2
LAW	4	4	3	2	1	3	1	2	0

count by 122 % and increase total distance of cabling by 100%, which increases cost and once again because of the increase of the number of paths this might cause overloading and delays during router table updates

2. Ring Topology

The Ring topology shown in figure (2) is an improvement over the Star topology. Besides having the advantages of the Star topology such as ease of management and configuration; it has some degree of scalability since it is easy to add another node. This topology provides some redundancy and therefore eliminates the single point of failure. All nodes have equal centrality and an alternate path through which they can be reached.

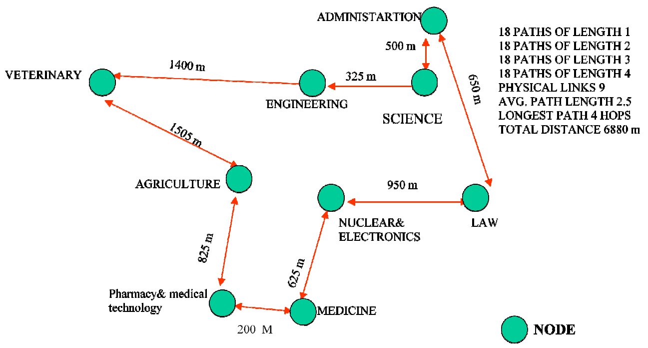

Figure (2) University Network Ring Topology

Applying the graph theory set of criteria to Ring topology reveals the following:

○ The average path length is 2.5, which quit long for a network of 9 nodes. Some nodes require 4 hops to reach each other.
○ There are a total of 72 possible paths available, which is the same as for Star topology, but will not overload the routing tables, nor cause delay during

Table (5) Average Path Length and Total Number of Geodesics

FACULTY	L1	L2	L3	L4	
VET	2	2	2	2	
AGR	2	2	2	2	
PH&MT	2	2	2	2	
MED	2	2	2	2	
N&E	2	2	2	2	
ENG	2	2	2	2	
ADM	2	2	2	2	
SCE	2	2	2	2	
LAW	2	2	2	2	
TOTAL	18	18	18	18	= 72

L1= PATHS OF LENGTH 1
L2= PATHS OF LENGTH 2
L3= PATHS OF LENGTH 3
L4= PATHS OF LENGTH 4

AVEG. PATH = 2.5

routing table updates.
○ Although this topology has an equal centrality throughout the network. This network reaches failure quickly because of its low redundancy.

The Ring topology can withstand one link failure and/or one node failure and still keep a connected network. Two simultaneous failures can lead to unreachable nodes due to low redundancy.

Although the Ring topology has a low centrality and some degree of redundancy, it has more physical links than Star topology, which means longer average path and a higher number of steps to reach some nodes.

If a single link fails in a Ring topology a 2-hop path can became a 7-hop path, which might cause congestion and results in delays, [1, 4].

If a second Ring is added to improve redundancy then the measures for this modified Ring topology is as follows:

Average path = 3.3; Hops = 4; No of geodesics = 540; Total cable distance = 13760 m

As it can be seen most measures did not improve. Average path has increased by 32%, available paths increased by 650% and total cabling distance increased by 100%. Although redundancy is improved, but at a high cost in terms of increased delay and actual cabling cost.

3. Partial Mesh Topology

This topology is the most difficult to build, since there is no simple rule to follow. If built incorrectly, this topology can have many of the disadvantages of the

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other discussed topologies without many of the benefits.

The method adopted to generate a Partial Mesh topology is based on Whitney's theorem [2], that say's

if a k-connected network is needed, then every node should have at least k links [2].

The procedure Starts by numbering the nodes at random. The randomization of this method allows for many topologies to be generated from same input data. Each node is associated with a number equal to the number of links needed at that node. This number is called the link deficit. Initially the deficit at each node is equal to the desired connectivity of the graph k. There are many algorithms to calculate the connectivity of a graph but k is normally taken as two or three [2].In our case to have adequate redundancy and optimize the generated network as far as graph theory measures are concerned, connectivity is selected to be 3. Connectivity of 1 will result in disconnected network and connectivity of 2 will result in a Ring network and hence minimum connectivity that can be used for this case is 3. The procedure proceeds by adding links one at the time until the deficit at each node is zero or less.

After generating 9 different topologies using the aforementioned Whitney's theorem, the partial mesh topology in figure (3) was selected from the generated topologies based on hop count, available paths (geodesics), average path and total distance covered.

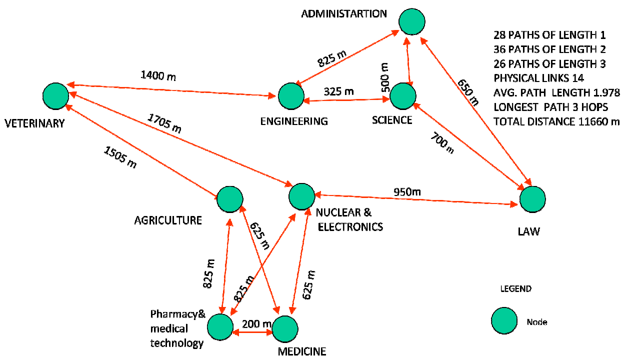

Figure (3) University Network Partial Mesh Topology (option 4 from Table 6)

A comparison table was developed, Table (6). From the table and by applying the aforementioned set of criteria to select the best topology the following can be noted:

1. All topologies have the same number of hop count, which is in this case 3, i.e. any node can reach any other node in at most 3 hops, which means that the diameter of the network still within the maximum limit of 7 hops and hence these topologies have a reasonable delay.
2. The hop count is a measure of the average path, which depends on the total number of paths (geodesics) and the number of paths for each count i.e. paths of length 1, length 2, and length 3. All topologies have nearly the same centrality i.e. these topologies are not dependent on any single or group of nodes which means that they are very resilient in the face of many local failures. This is an indication of reasonable redundancy where the link failure does not segregate the network and nodes can still communicate. Node failure will not damage the network and a part from the failed node; rest of network will function normally.

From table (6) it can be seen that option 4 has the lowest number of geodesics and option 6 has the lowest average path length.

Looking at the total covered distance it can be seen that option 4 & option 7 have the lowest total distance of 11660 m.

Based on the above discussion option 4 satisfies all the criteria and therefore was selected as the topology to be compared with Star and Ring topologies.

Table (6) Comparison between the Generated Partial Mesh Topologies

TOPOLOGY	**Option**								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Physical. Links	14	14	14	14	14	14	14	14	14
Length type 1	28	28	28	28	28	24	28	28	28
Length type 2	48	30	36	36	41	42	30	42	48
Length type 3	26	35	40	26	31	23	36	36	24
No. of geodesics	102	93	104	90	100	93	94	106	100
Average Path	1.98	2.075	2.115	1.978	2.03	1.946	2.085	2.076	1.96
Hops	3	3	3	3	3	3	3	3	3
Total Distance	12985	11860	13235	11660	12230	12430	11660	12230	13235

V. Analytic Hierarchy Process (AHP) method

Having decided on the Partial Mesh topology that to be included in the comparison with the other two topologies, namely Star and Ring.

Table (7) was developed based on the previously discussed graph theory criteria.

From table (7) it can be seen that Star and Ring are better choice than P. Mesh when considering the number of the geodesics for each topology. But Star and P Mesh topologies are the better choice if considering average path length and/or number of hops. The P. Mesh is outstanding winner when it comes to redundancy and has nearly equal centrality with the

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Ring topology. The Star topology fails measurably when it comes to centrality and redundancy.

Therefore to decide which of the three is the better choice on all accounts a management tool known as the Analytic Hierarchy Process (AHP) is used. This method is designed to solve problems involving multiple criteria.

The process requires the decision maker to provide judgment about the relative importance of each criterion and then specify a preference on each criterion for each decision alternative. The output of the AHP prioritized ranking indicating the overall preference for each of the decision alternatives [5].

Table (7) Comparison between Partial Mesh, Ring and Star Topologies

TOPOLOGY	STAR	RING	P.MESH option 4
Physical Links	8	9	14
No. of geodesics	72	72	90
Average Path	1.78	2.5	1.978
Hops	2	4	3
T. Distance	7730	6880	11660
Centrality	High	Low	Low
Redundancy	V.Low	Low	High

The AHP process can be summarized in the following steps:

- First step is to develop the hierarchy model for the problem in terms of the overall goal, criteria and decision alternatives. In our case the hierarchy for network topologies is shown in figure (4).

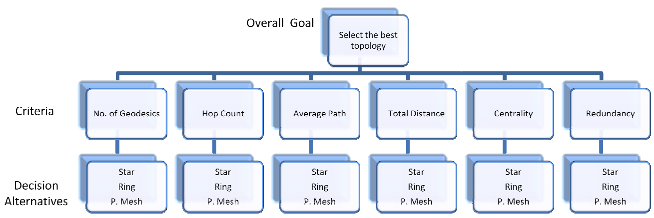

Figure (4) AHP hierarchy model

- Develop pair-wise comparison matrix: this matrix is developed for each criterion using the decision alternatives in order to develop priorities for those alternatives. Entries of the matrix are taken from a pair-wise comparison scale for AHP preferences as can be seen in table (8).

- Synthesizing judgment: this is a procedure for calculating the priority for each of the elements being compared in our case the three topologies.

This is done through the following steps:

1. Sum the values in each column of the pair-wise comparison table.

2. Divide each element in the pair-wise comparison matrix by its column total, the resulting matrix known as the **normalized pair-wise comparison matrix**.

3. Compute the average of the elements in each row of the normalized matrix. These averages provide an estimate of the relative priorities of the elements being compared.

4. Estimate consistency ratio, which is a measure of the consistency of pair-wise comparison judgment. This ratio must not exceed 0.1 otherwise values in the pair-wise comparison matrix have to be revised [5].

Table (8) a pair-wise comparison scale

Verbal judgment of preference	Numerical Ratings
Extremely preferred	9
Very strongly to Extremely preferred	8
Very strongly preferred	7
Strongly to very strongly preferred	6
Strongly preferred	5
Moderately to strongly preferred	4
Moderately preferred	3
Equally to moderately preferred	2
Equally preferred	1

The consistency ratio procedure is summarized in the following steps:

1. Multiply each value in each column of the matrix by the relative priority of the corresponding item considered. This process is continued until all columns are done. The values across the rows are summed to obtain a vector of values labeled as weighted sums.

2. The elements of the vector of weighted sums are divided by the corresponding priority value.

3. The average of the values obtained in step 2

is computed. This average is denoted by λ_{max}.

4. The consistency index (CI) is computed, which defined as:

$$CI = \frac{\lambda max - n}{n-1}$$

n = number of elements being compared.

The consistency ration (CR) is computed

$$CR = \frac{CI}{RI}$$

RI is the random index.

This is the consistency index of a randomly generated pair-wise comparison matrix and is found to be as in the table (9).

Table (9) AHP Random Index

n	3	4	5	6	7	8
RI	.58	.90	1.12	1.24	1.32	1.41

Table (10) was developed from the pair-wise calculations for the three topologies and indicates that the Partial Mesh topology is the best choice, Ring comes in the second place, and the third place is for the Star topology.

As it was mentioned earlier The Star topology suffers from very serious disadvantage and that is the high centrality, which results in a very high risk of failure and could be a very damaging bottleneck.

The Partial Mesh Figure (3) is the best of the three compared topologies.

In addition to satisfying the applied criteria, it has a better redundancy than the other two, which allows the network to function even if there is a failure in a link or a node.

Table (10) Overall Priorities

Alternative	**Overall priority**
P. Mesh	0.442
Ring	0.328
Star	0.230

VI. Conclusions

Using graph theory to design computer core networks can help in choosing the most appropriate topology to use for the network and by the aid of the AHP procedure will assist the network designer to confirm his decision on the type of topology to adopt.

VII. References

1. Valdis Kerbs , "The social life of routers, applying knowledge of human networks to the design of computer networks", the internet protocol journal, Cisco publications, IPJ issues. 2001, http/ www.cisco.com
2. Andrews S. Tanenbaum, "Computer networks", prentice / hall international edition, 1981
3. "Designing packet service internetworks", Cisco Systems Inc., 2002 http//www.cisco.com/univercd/cc/td/doc/cinsintwk/idg4/nd2009.htm
4. "Network Essentials, hands on, self-paced training for supporting local and wide area networks", Microsoft press, 1996
5. Anderson Sweeney and William West, "An introduction to management science quantitive approaches to decision making" , seventh edition, 1994